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Cosmology in scalar tensor theory and asymptotically de-Sitter Universe

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We have investigated the cosmological scenarios with a four dimensional effective action which is connected with multidimensional, supergravity and string theories. The solution for the scale factor is such that initially universe undergoes a decelerated expansion but in late times it enters into the accelerated expansion phase. Infact, it asymptotically becomes a de-Sitter universe. The dilaton field in our model is a decreasing function of time and it becomes a constant in late time resulting the exit from the scalar tensor theory to the standard Einstein's gravity. Also the dilaton field results the existence of a positive cosmological constant in late times.

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The observed location of the first acoustic peak of CMB temperature fluctuation corroborated by the latest BOOMERANG and MAXIMA data [1] favours a spatially flat matter dominated universe whose energy density is dominated by some *missing energy component*. On the other hand, recent measurements of the luminosity-redshift relations observed for a number of newly discovered type Ia supernovae indicate that at present the universe is expanding in an accelerated manner suggesting that this *missing energy* has a negative pressure [2]. The first and obvious choice for this missing component is the vacuum energy or cosmological constant Λ. However this possibility of Λ being the dominant component of the total energy density of the universe, has problem that the energy scale involved is lower than normal energy scale of most particle physics model by factor of $\sim 10^{-123}$. The next choice for this missing energy is a dynamical Λ in the form of a scalar field slowly rolling down a considerably flat potential so that its slowly varying energy density mimics an effective cosmological constant. This form of missing energy density is called *Quintessence* and this is similar to the inflation field with the difference that it evolves in a much low energy scale.

There are a number of quintessence models which have been put forward in recent years and most of them involve a minimally coupled scalar field with different type of potentials dominating over the kinetic energy of the scalar field giving rise to an effective negative pressure [3]. Although all these models have their own merits in explaining the missing energy and the accelerated expansion of the universe, they also have some drawbacks. First of all the minimally coupled self interacting scalar field models will be ruled out if the observations predicts that the missing component of the energy density obeys an equation of state $p = \gamma\rho$ with $\gamma < -1 (\rho \geq 0)$, and this sort of equation of state is in reasonable agreement with different observations [4]. Also the inequality $dH^2(z)/dz \geq 3\Omega_{m0}H_0(1+z)^2$ should have to be satisfied for minimally coupled scalar field and its violation will certainly point towards a theory of non Einstein gravity such as scalar tensor theories where the scalar field is nonminimally coupled to gravity.

Also if one considers a supergravity theory in higher dimensions, then gravity is in general coupled to gauge fields composing the bosonic sectors. The reduction to four dimensions leads to a nontrivial coupling between the gravity and scalar fields, called dilaton, some of which come from the compactifications of the internal dimension. The cosmological scenarios of these effective models in four dimensions has been extensively studied [5]. In recent years there are also attempts in modelling the missing energy of the universe and to explain its late time accelerated expansion in perview of these scalar tensor theories where the scalar field is nonminimally coupled to gravity sector [6]. Attempts have also been done to study the late time accelerated expansion of the universe in context of original Brans-Dicke (BD)theory where the BD scalar field is massless [7,8]. One of the problems in these models is that it is difficult to incorporate the decelerating expansion phase of the universe in these models. Hence these models are always accelerating which seriously contradicts with the big bang nucleosynthesis and structure formation scenario of the universe. It was shown by Banerjee and Pavon [7] that one can avoid such problem by assuming the BD parameter to be a function of the BD scalar field ϕ. Also all these scalar tensor theories are labelled by a parameter ω and another serious problem is that in most of these models, in order to have the accelerated expansion of the universe in late time together with the right estimates for Ω_m and Ω_ϕ, the range of this parameter ω is too small to match with the solar system experimental bound $\omega > 500$. Bertolami and Martins [6] have obtained the solution for the accelerated expansion of the universe in BD cosmology with a ϕ^2 potential with large $|\omega|$. But in their work they have not considered the positive energy conditions for the matter and

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the scalar field. The reasons for these problems may be that in most of these models the simple power law expansion of the universe is assumed. Hence it may be worthwhile to study some different expansion behaviour of the universe which can give rise to late time accelerated expansion of the universe and also can solve the problems described above.

In the present work, we have used the low energy four dimensional effective action for the higher dimension theory, together with a matter fluid having a dissipative pressure over and above its positive equilibrium pressure. The CDM is in general considered to be a perfect fluid having a zero positive equilibrium pressure. However it has been proposed recently that the CDM must self interact in order to explain the detailed structure of the galactic halos [9]. This self interaction may create a pressure for the CDM which can also be negative [10]. Also, as demonstrated in a recent paper by Zimdahl et.al [11], one can have a pressure in the CDM if there exists an interaction which does not conserve particle numbers, and negative pressure can exist due to particle production out of gravitational field. In this case, the CDM is not a conventional dissipative fluid, but a perfect fluid with varying particle number. Zimdahl et.al have shown that even extremely small particle production rate can cause the sufficiently negative pressure to violate the strong energy condition. In our calculations, we have not assumed any particular model for the negative pressure of the CDM rather we have studied its effect in the late time expansion of the universe. Unlike the previous works on nonminimally coupled scalar tensor theories, we have not assumed any particular form for the potential for the BD scalar field. Instead we have assumed the temporal dependence of the BD scalar field in such way that one can recover the Einstein's gravity in present day. This will ensure that there is no conflict with the present day Solar system experimental results which are very much consistent with the Einstein's gravity. The solution for the scale factor of the universe is such that initially the universe undergoes decelerated expansion which is necessary for the structure formation scenario in the matter dominated universe, but in late time it enters the accelerated expanding phase suggested by the recent Supernova observations. In fact it has been shown that one can get a de-Sitter expanding universe asymptotically. The potential we obtain for such solution is a combination of terms like $\phi^p(ln(\phi))^q$ with p and q taking different values. This kind of potential has earlier been studied for inflationary model with minimally coupled scalar field by Barrow and Parson [12]. Terms such as this also appears in the Coleman-Weinberg potential for new inflation [13]. The behaviours of energy densities of the matter and the dilaton field are shown to be quite satisfactory in our model.

The field equations derived from the low energy effective action of the string theory is given by

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{\phi} + \frac{\omega}{\phi^2}(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}) + \frac{1}{\phi}[\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi] - g_{\mu\nu}\frac{V(\phi)}{2\phi}, \tag{1}$$

where $T_{\mu\nu}$ represents the energy momentum tensor of the matter field, ϕ is the dilaton field and ω is a dimensionless parameter. We have assumed the matter content of the universe to be composed of a fluid represented by the energy momentum tensor

$$T_{\mu\nu} = (\rho + P)v_\mu v_\nu + Pg_{\mu\nu}, \tag{2}$$

where ρ and P are the energy density and effective pressure of the fluid respectively and $v_\mu$is the four velocity of the fluid i.e, $v_\mu v^\mu = -1$. The effective pressure of the fluid includes the thermodynamic pressure p and a negative pressure π, which could arise either because of the viscous effect or due to particle production, i.e,

$$P = p + \pi \tag{3}$$

The dynamics of the dilaton field is governed by the equation,

$$\Box\phi = \frac{T}{2\omega+3} + \frac{1}{2\omega+3}\left(\phi\frac{dV(\phi)}{d\phi} - 2V(\phi)\right) \tag{4}$$

where T is the trace of $T_{\mu\nu}$. The background metric is considered to be standard Friedman-Robertson- Walker one with the signature convention $(-,+,+,+)$ and R is the scale factor. We restrict ourselves for spatially flat metric only. We work in Jordan frame. One interesting thing about working in Jordan frame is that the conservation equation holds for matter and scalar field separately. Or in a slightly different way, the Bianchi Identity along with the wave equation (4) gives the matter conservation equation

$$\dot{\rho} + 3\frac{\dot{R}}{R}(\rho + p + \pi) = 0 \tag{5}$$

Expressing explicitly in terms of the FRW metric the field equations and the wave equation respectively appear to be

$$3\frac{\dot{R}^2}{R^2} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{V}{2\phi} = \frac{\rho}{\phi}, \tag{6}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{\ddot{\phi}}{\phi} + 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{V}{2\phi} = -\frac{p}{\phi} \tag{7}$$

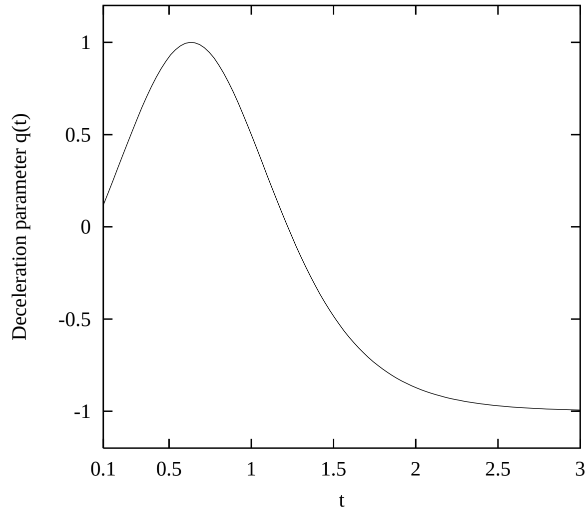

FIG. 1. The deceleration parameter q *vs* time t in units of 10 Gyrs

$$\ddot{\phi}+3\frac{\dot{R}}{R}\dot{\phi}=\frac{\rho-3p}{2\omega+3}-\frac{1}{2\omega+3}\left[2V-\phi\frac{dV}{d\phi}\right] \tag{8}$$

In this situation we are at liberty to make some assumptions as we have more unknowns $(R, \phi, \rho, p, \pi, V)$ with lesser numbers of equations to determine them. Unlike the previous investigations where people have assumed the power law expansion of the universe together with some suitable form of the potential, we will assume the form for the dilaton field in such a way so that it stabilises quickly with time resulting the transition to GR. Subsequently we calculate the form of the potential which can give rise to this kind of behaviour.

We assume the ansatz

$$\frac{\dot{\phi}}{\phi}=-\beta Exp[-\alpha(t-t_0)] \tag{9}$$

and

$$H=\frac{\dot{R}}{R}=H_1 Exp[-\alpha(t-t_0)]+H_2 \tag{10}$$

where $H_1,\ H_2,\ \alpha,\ \beta$ and t_0 are all positive constants. This essentially means that we are working in a scenario where the scale factor and the scalar field evolve as

$$R=R_0 Exp[H_2 t-\frac{H_1}{\alpha}e^{-\alpha(t-t_0)}] \tag{11}$$

and

$$\phi=\phi_0 Exp[\frac{\beta}{\alpha}e^{-\alpha(t-t_0)}] \tag{12}$$

As is evident from this expression, the dilaton field is an exponentially decreasing function of time and quickly stabilises at some constant ϕ_0 for large time. Hence in late time one could recover General Relativity by identifying $1/\phi_0$ with G_0, the present Newtonian constant. Hence whatever be the choice of ω we have in our model. that really does not contradict the solar system bound on ω as for the present day the theory becomes GR.

The deceleration parameter seems to be an important parameter to specify the expansion of the universe. For last several decades the standard cosmological models favoured a presently matter dominated universe expanding in a decelerated fashion. The positive value of the deceleration parameter was more or less compatible with all cosmological tests. But the recent observation seem to favour negative value of this parameter to portray an accelerated expansion of the universe. With an ansatz of the form given by (10), the deceleration parameter q is given by

$$q=-\frac{\dot{H}}{H^2}-1=\frac{\alpha H_1 e^{-\alpha(t-t_0)}}{[H_2+H_1 e^{-\alpha(t-t_0)}]^2}-1 \tag{13}$$

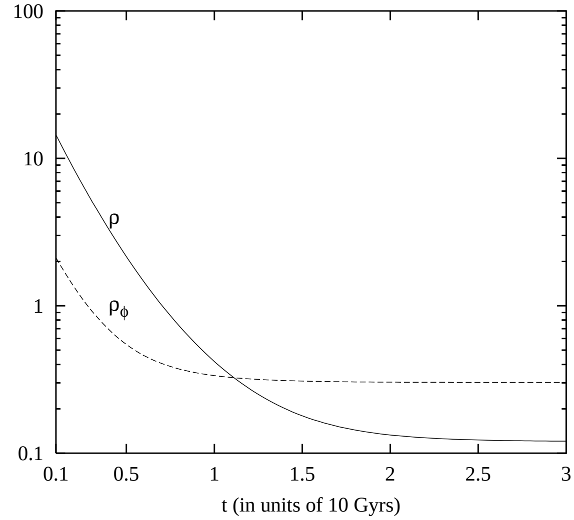

FIG. 2. Energy densities ρ and ρ_ϕ *vs* time t

The behaviour of the deceleration parameter q is presented in figure 1. For this we have chosen the parameter t_0 to be 10^{10} yrs and also we have set our initial time to 10^9 yrs, which is approximately the time for first bound structures to be formed [14]. The graphical representation of q shows that initially it has a positive value suggesting decelerated expansion, but with time it attains a negative value depicting accelerated expansion and finally saturates to -1. This feature of q quite satisfactorily explains the evolution of the universe where the matter dominated universe decelerates but starts accelerating at late time as suggested by present observation [2]. It is interesting to notice that the universe starts accelerating very recently which is quite consistent with present estimated age of the universe [15]. Also, the figure shows that it asymptotically the expansion becomes de-Sitter suggesting the existence of a positive cosmological constant. Apart from the appropriate form of evolution of the universe, constraints are imposed on an acceptable model from the ratio of the energy densities of matter and scalar field. In the early times, such as at 10^9 yrs, the matter energy density dominates the total energy density to justify the structure formation scenario, whereas at late time the scalar energy density plays the predominant role to explain the so called missing energy density of the universe.

To find whether the present ansatz accommodates these constraints we find the energy densities of the matter and the scalar field. With this specific kind of evolution of the universe [(10),(12)] the energy density of the matter and the scalar field turns out to be respectively

$$\rho = \left\{ \frac{3H_2}{8}(16H_1 - 9\beta)e^{-\alpha(t-t_0)} - 3H_1[\beta(1+\omega) + H_1]e^{-2\alpha(t-t_0)} \right\} \phi_0 Exp[\frac{\beta}{\alpha} e^{-\alpha(t-t_0)}] + \rho_0 \tag{14}$$

and

$$\rho_\phi = \left\{ 3H_2^2 + \frac{27}{8}\beta H_2 e^{-\alpha(t-t_0)} + 3H_1[\beta(1+\omega) + 2H_1]e^{-2\alpha(t-t_0)} \right\} \phi_0 Exp[\frac{\beta}{\alpha} e^{-\alpha(t-t_0)}] - \rho_0. \tag{15}$$

where ρ_0 is an integration constant with the choice

$$\omega + 1 = -\frac{(16H_1 - 3\beta)^2}{8\beta(16H_1 - \beta)} \quad \text{and} \quad \alpha = 8H_2. \tag{16}$$

In order to ensure the positive energy condition for both matter and scalar field the constraints on the constants are $(16H_1 - 3\beta)^2 > 8H_1(16H_1 - \beta)$ and $16H_1 - 9\beta > 0$. In figure 2 we plot the the energy densities of both the matter and the scalar field. The energy scale is plotted in units of 10^{-47} GeV^4. We find that the above constraints are accommodated i.e, the matter energy dominates the scalar energy in the early time but it evolves at such a rate that at late time the scalar energy density takes over the matter density and finally tracks to a constant ratio of the two densities. This is a very important behaviour so far as the late time acceleration and quintessence scenario are concerned. This tracking behaviour is a possible solution to the cosmic coincidence problem of the quintessence proposal. An interesting point to note here is that the energy density of matter $\rho^{1/4}$ is $\sim 10^{-3}$ eV at an epoch 1 Gyrs when the first bound structures were formed, which agrees quite well with the available data [14]. On the other hand the density parameters also gives a clear picture about the ratio of energy densities of the universe at different epochs. A plot of the density parameter of matter ($\Omega_m = \frac{\rho}{3H^2\phi}$) and that of scalar field ($\Omega_\phi = \frac{\rho_\phi}{3H^2\phi}$) is shown in

figure 3. Ω_m dominates the Ω_{tot} in the early time, but off late Ω_ϕ becomes the major contributor of Ω_{tot} and finally both Ω_m and Ω_ϕ saturates at values 0.28 and 0.72 respectively, which match the recent observations excellently [2].

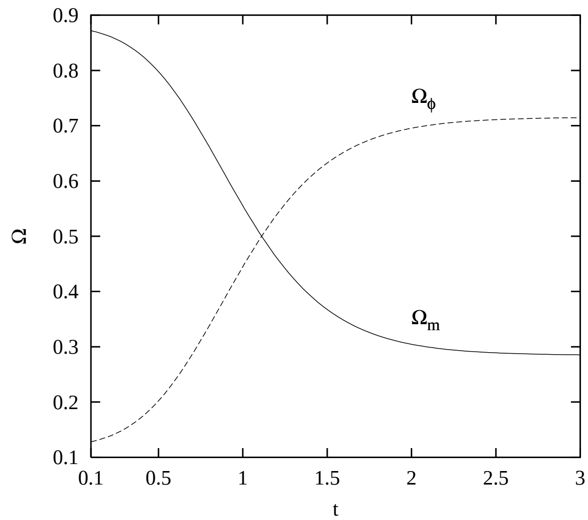

FIG. 3. Density parameters Ω_m and Ω_ϕ *vs* time t

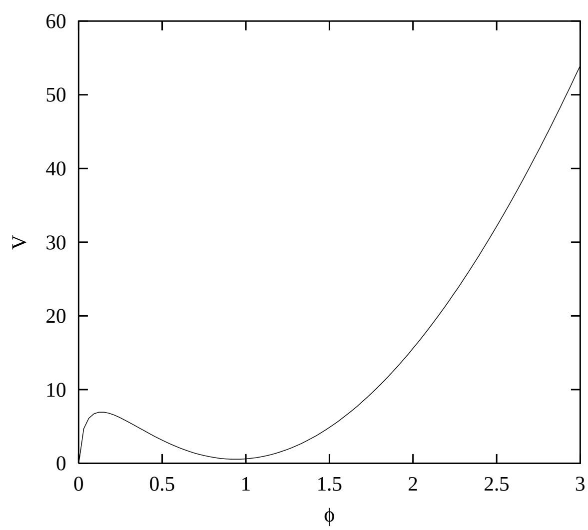

FIG. 4. Plot of potential

The form of potential admissible in this ansatz is given by

$$V(\phi) = \left(\frac{8H_2}{\beta}\right)^2 [3H_1(\beta\omega + 2H_1) - \frac{\omega}{2}\beta^2]\phi \left[\ln\frac{\phi}{\phi_0}\right]^2 + 6H_2^2\phi\ln\frac{\phi}{\phi_0} + 6H_2^2\phi - 2\rho_0 \tag{17}$$

This kind of potential can arise due to higher order loop corrections of the Coleman-Weinberg potential. This type of potential had been used earlier in quite a few references, specially in context of inflationary scenario [7]. We have plotted the potential in figure 4. An interesting point to note here is that the minimum of the potential has a nonzero value at which the dilaton stabilises. This asymptotically generates a positive cosmological constant which provides a good enough explanation for the missing energy component. We have also plotted in figure 5, the equation of state for the dilaton field, where one finds that the equations of state stabilises at -1 in late times suggesting that the dilaton field gives rise to a positive cosmological constant in late time.

The effective pressure of the CDM fluid is given by

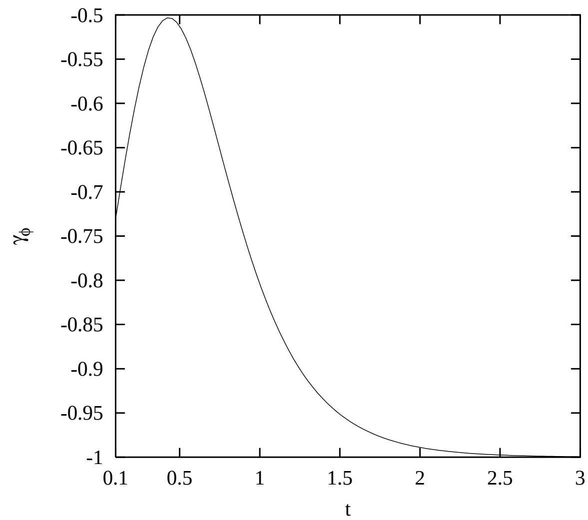

FIG. 5. Plot of the equation of state for the scalar field γ_ϕ *vs* ϕ

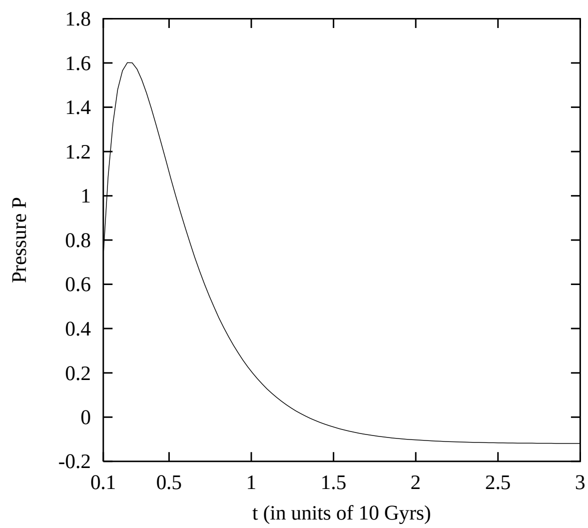

FIG. 6. Plot of the effective pressure P

$$P = \left\{ e^{-\alpha(t-t_0)} \left[\frac{5}{8} H_2 (16H_1 - 9\beta) \right] - e^{-2\alpha(t-t_0)} (\beta - 3H_1)[\beta(1+\omega) + H_1] \right\} \phi_0 Exp[\frac{\beta}{\alpha} e^{-\alpha(t-t_0)}] - \rho_0 \quad (18)$$

In figure 6 we have plotted the effective pressure P of the CDM. It has shown that the CDM has very small effective negative pressure in late time. This is similar to what Chimento et.al [10] and also Zimdahl et.al [11] assumed in their investigations.

In conclusion, we have studied the cosmological evolution with the low energy effective string action. We have assumed the form of the dilaton field in such a way so that it stabilises quickly with time resulting the transition to the GR theory. Hence the problem of having a higher value of the parameter ω consistent with Solar system observation is no more in our theory which is present in most of the quintessence models in scalar tensor theories studied in recent times. Moreover the scale factor of the universe in our model is such that one can have a decelerating universe in early time but in late time the universe becomes accelerating which is consistent with the recent supernovae observations. In fact the universe asymptotically becomes a de-Sitter universe. We have also calculated the form of potential that can give rise to such kind behaviour. It has been shown that the potential has a non zero minima where the dilaton will stabilise and this will give give rise to a positive cosmological constant asymptotically. The behaviours of the energy densities of the matter and scalar field show that although in early time, the energy density of the matter field is greater than that of the scalar field but in late times the scalar field will dominate explaining the missing energy component. We have fixed our initial time at 1 Gyr. But initially the universe is decelerating as well as the matter energy density dominates over that of the scalar field, hence one can extend our model back in earlier time to meet the constraints from the big-bang nucleosynthesis.

It seems that this model can be a viable model, at least theoretically, to explain the late time acceleration of the universe and the missing energy component. It will be interesting to see whether this model actually matches with Type Ia supernovae observations and also with CMBR experiments like BOOMERANG and MAXIMA.

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